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Scalar meson exchange and the baryon spectra

P. Stassart and Fl. Stancu
Université de Liège, Institut de Physique, B 5, Sart Tilman, B-4000 Liège 1, Belgium

J.-M. Richard
Institut des Sciences Nucléaires, Université Joseph Fourier–IN2P3-CNRS,
53, avenue des Martyrs, F-38026 Grenoble Cedex, France
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Abstract

We explore the role of a scalar meson exchange interaction between quarks in a semirelativistic constituent quark model where the quarks are also subject to a linear confinement. We search for a variational solution and study how the spectrum evolves when the strength of the scalar meson exchange increases. Our results have good implications on the description of the low-lying baryon masses, especially on the long-standing problem of the relative position of the first positive-parity (Roper resonance) and the first orbitally excited states.

It has been suggested [1,2] that for energies below the scale of spontaneous chiral symmetry breaking, nonstrange and strange baryons can be viewed as systems of three quarks interacting via exchange of Goldstone bosons (pseudoscalar mesons). It has also been shown that a properly parametrized interaction provides good baryon spectra with a correct order of positive and negative parity levels both in a nonrelativistic [3] or in a semirelativistic [4] treatment. In both parametrizations, the pseudoscalar meson exchange interaction has two distinct parts: a long-range Yukawa potential tail and a short-range part having opposite sign as compared to the Yukawa potential tail. It is the latter which plays a major role in describing the baryon spectra in the frame of Goldstone boson exchange (GBE) models.

The underlying symmetry of the GBE model is related to the flavour–spin $\mathrm{SU_F}(3) \times \mathrm{SU_S}(2)$ group combined with the $\mathrm{S_3}$ symmetry required by a system of three identical quarks. A thorough independent analysis of this model, recently performed for $L = 1$ baryons, has shown [5] that within the chiral picture one can obtain more satisfactory fits to the observed spectrum than with one-gluon exchange (OGE) models.

Although the models [2–4] are thought to be a consequence of the spontaneous chiral symmetry breaking, the chiral partner of the pion, the σ-meson, is not considered explicitly. One can think of having mocked up its contribution in the parameters of the Hamiltonian [3,4], e.g., in the regularisation parameter of the short-range spin–spin term. The price

which could have been paid is the large role played by the η' meson exchange which comes into the interaction with a strength $g_0^2/(4\pi)$ about equal [4] or larger [3] than the strength $g_8^2/(4\pi)$ associated with the pseudoscalar octet (π, K, η).

Meson exchange potentials including σ-exchange have been considered in the so-called "hybrid" models where both meson and gluon exchanges contribute to the quark–quark interaction. However these studies were mainly devoted to the baryon–baryon interaction [6,7] and little conclusion has been explicitly drawn about the role of the σ-exchange to the baryon spectrum. Moreover these models are all nonrelativistic.

Here we study the explicit role of a σ-exchange interaction by considering the following semirelativistic Hamiltonian

$$H_0 = \sum_i \left(m_i^2 + p_i^2\right)^{1/2} + \frac{1}{2}\sqrt{\sigma}\sum_{i<j} r_{ij} - \frac{g_\sigma^2}{4\pi}\sum_{i<j}\left[\frac{\exp(-\mu_\sigma r_{ij})}{r_{ij}} - \frac{\exp(-\Lambda r_{ij})}{r_{ij}}\right], \tag{1}$$

where $r_{ij} = |\vec{r}_{ij}| = |\vec{r}_i - \vec{r}_j|$. The second term is the confinement potential with a string tension [8]

$$\sqrt{\sigma} = 1 \text{ GeV fm}^{-1}. \tag{2}$$

The third term is the σ-exchange interaction, the form of which corresponds to the Pauli–Villars regularisation. In this term we fixed the σ-meson mass to $\mu_\sigma = 600$ MeV. The coupling constant $g_\sigma^2/(4\pi)$ and the regularization parameter Λ are taken as variable parameters. The interaction is attractive inasmuch as we view σ as a pair of correlated pions, as in the nucleon–nucleon interaction. In practice this is achieved whenever $\mu_\sigma < \Lambda$.

From the pion–nucleon coupling constant $g_{\pi NN}^2/4\pi \simeq 14$, one obtains a pion–quark coupling constant $g_{\pi qq}^2/(4\pi) = 0.67$ which has been used in [3,4]. Assuming a σ–nucleon coupling $g_{\sigma NN}^2/4\pi \simeq 8$ [9] one obtains, by scaling, $g_{\sigma qq}^2/4\pi \simeq 0.4$. We found that the values used in the literature [6,7] are larger. A recent evaluation [10] of the σ–quark coupling constant from the two-pion exchange interaction between constituent quarks leads to $g_{\sigma qq}^2/4\pi \simeq 1$. In the following we study the role of the σ-meson exchange for various ranges of $g_{\sigma qq}^2/(4\pi)$ as shown below.

We search for a variational solution of the Hamiltonian (1). Its general form is

$$\psi_n\left(\vec{r}_{12}, \vec{r}_{23}, \vec{r}_{31}\right) = \prod_{i<j} f\left(r_{ij}\right)\phi_n\left(\vec{r}_{12}, \vec{r}_{23}, \vec{r}_{31}\right). \tag{3}$$

where f is an orbital two-body correlation function and ϕ_n with $n \neq 0$ includes orbital excitations [11]. The function f is parametrized as

$$f(r) = r^\delta \exp\left\{-W(r)\,\gamma r - [1 - W(r)]\,\gamma' r^{1.5}\right\}, \tag{4}$$

with

$$W(r) = \frac{1 + \exp(-r_0/a)}{1 + \exp[(r - r_0)/a]}. \tag{5}$$

The quantities γ, γ', a, r_0 and δ are variational parameters.

It is instructive to discuss the limit $\Lambda \to \infty$ first. In this case the second term in the σ-exchange interaction vanishes. Such a simplified form can be justified through arguments related to the differences appearing in the scalar and pseudoscalar propagators [12] in a Nambu–Jona-Lasinio model. From there one can argue that the Fourier transform would lead to a Yukawa *plus* a contact term for a scalar exchange, and a Yukawa *minus* a contact term for a pseudoscalar exchange. The latter is consistent with Refs. [3,4]. Due to the addition of the long and short range contributions in the scalar case, the two contributions can be combined in a single attractive term. In the minimization procedure for $\Lambda \to \infty$, we found that the ground-state expectation value of (1) varies smoothly with all parameters but δ. This is quite natural because δ cares for the behaviour of the wave function near the origin, typical for the solution of a relativistic equation with a singular potential. The dependence of the first three expectation values of (1) on the coupling constant is displayed in Fig. 1. Alternatively, Table I shows the energies of the first excited states E_{1p} and E_{2s} for a ground state energy E_{1s} set to 940 MeV. One can see that the energy difference between the first radially excited state and the negative parity state is positive up to the coupling constant value of 0.20 but tends to vanish. Note also that the difference between the radially excited state and the ground state remains practically constant as a function of the coupling constant, while the mass difference between the orbitally excited state and the ground state increases with $g_\sigma^2/(4\pi)$. The reason for this is that at $g_\sigma^2/4\pi \neq 0$, every s state is lowered with respect to the p states. The wave function of the latter is small around the origin, so that it reduces some of the attraction in the expectation value.

Our results show that the mass difference $E_{2s} - E_{1p}$ becomes negative for $g_\sigma^2/4\pi \gtrsim 0.20$. This is precisely the desired behaviour for reproducing the correct order of the experimental spectrum, as in Ref. [3]. These calculations indicate that a model incorporating a potential, the Laplacian of which is negative in a region around the origin, can yield the right ordering of the lowest positive and negative parity states. For the potential of Eq. (1) such a situation is achieved for suitable values of the coupling constant $g_\sigma^2/(4\pi)$. Indeed the sign of the Laplacian governs the relative magnitude of radial vs. orbital excitations, as shown rigorously in the two-body case [13] and approximately in the three-body case [14].

Realistically we expect Λ to be finite. We illustrate this case by taking $\Lambda = 2$ GeV, consistent with the cut-off used in the Bonn nucleon–nucleon potential [9]. In this case, we let the coupling constant vary up to 1.34, the latter being twice the value of the pion coupling constant. Fig. 2 shows that the expectation value of the $L = 1$ state decreases slightly slower than that of the first radially excited state. The latter remains practically parallel to the expectation value of the ground state.

It is important to understand the effects incorporated in the Hamiltonian (1). For the sake of the discussion, we introduce the quantity

$$R = (E_{1p} - E_{1s})/(E_{2s} - E_{1s}) \tag{6}$$

where E_{1s}, E_{1p} and E_{2s} are the expectation values for the ground state, the first negative parity state and the first radially excited state, respectively. So that if $R \gtrsim 1$, the desired order is obtained. It is well known that the "naive" potentials of harmonic oscillator type have $R < 1$ (see e.g. [15]), and no tuning could improve the situation. It is also known that the addition of the OGE interaction with reasonable strength does not improve the

situation either, the Roper resonance always appearing above the first negative parity states, in contradiction to the experimental situation. The fact that $R < 1$ for "naive" potentials is not surprising, at least for the 2-body problem. There are general theorems stating that R is smaller than unity if $\triangle V \geq 0$ [13]. This happens for most empirical potentials and in particular for the celebrated Coulomb + linear potential of heavy quarkonia. Although there do not exist rigorous theorems in the 3-body case, within the approximation of the lowest hyperspherical partial wave, one can also show that $R < 1$ if $\triangle V \geq 0$. [14].

Actually we studied in detail several distinct effects contributing to the position and ordering of levels:

i) In our potential we introduce components with $\triangle V \leq 0$ through the Yukawa-type potential. This is the case of Fig. 1 or Table I. When a regularizing term with a finite Λ is subtracted from the Yukawa type potential it is not surprising that R decreases because the regularization term leads to a $\triangle V \geq 0$ contribution at small values of r.

ii) We adopt a relativistic kinematics. In the 2-body case (Herbst equation) we find that R is closer to unity with a relativistic kinematics than if we treat the same problem nonrelativistically (NR). For clarity in Table II we show results obtained in the 2-body case with the interaction

$$V = \lambda r - g \exp(-\mu r)/r, \tag{7}$$

where the parameters λ, g and μ are taken in units of the quark mass m. One can see that with or without Yukawa-type potential, the ratio R is systematically larger in the semirelativistic case than in the nonrelativistic case. The case $\lambda = 0.01$ tests that relativistic effects disappear in the weak-coupling limit.

iii) We looked comparatively at the 2- and 3-body cases to see if there are differences in the value of R. In the NR case it was shown [14] that R is very similar for the 2-body and the 3-body problem for potentials of type $V \propto r^\beta$ with $\beta \geq -1$. It occurs however that R might be larger in the 3-body case than in the 2-body one when the potential V is such that $\triangle V \leq 0$. In our case, this is more delicate. We have a linear term with a positive Laplacian, and a Yukawa term with a negative Laplacian. The latter being however of short-range character, it is less easily felt by baryons which, in first a approximation, obey a two-body dynamics with an effective angular momentum $J = 3/2$ [14]. For instance, for the non-relativistic version of our Hamiltonian (1) with $m_i = 1$, $\sqrt{\sigma} = 2$, $g_\sigma^2/(4\pi) = 0.2$, $\mu_\sigma = 1$ and $\Lambda = \infty$, the ratio R is found equal to $R \simeq 0.67$ for the 3-body case, to be compared to $R \simeq 0.60$ the 2-body case.

An interesting finding is that the three effects above, $\triangle V \leq 0$, the relativistic kinematics and the 3-body effect cumulate together and lead to the result shown in Tables I and II.

A correct level ordering of the first positive parity and the first orbitally excited states has been achieved in Refs. [3,4] not at the level of the spin-independent Hamiltonian — as discussed above — but through adding the GBE interaction. As mentioned in the beginning of this study, this interaction contains flavour–spin operators, i.e., it is related to the $\mathrm{SU_F}(3) \times \mathrm{SU_S}(2)$ group. In a simpler version, one can use $\mathrm{SU_I}(2)$ instead of $\mathrm{SU_F}(3)$. Then one can easily understand the role of the GBE interaction by considering the following operator

$$O^{IS} = -\sum_{i\,<\,j} \vec{\tau}_i \cdot \vec{\tau}_j \; \vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{8}$$

for a three-quark system. The effect of this interaction is similar to that shown in Fig. 1 as suggested by Table III where three expectation values of O^{IS} are indicated. One can see that the ground state and the first radially excited state, both of orbital symmetry $[3]_O$ are lowered by 15 units while the first negative parity state of either spin 1/2 (i.e., $[21]_S$) or 3/2 (i.e., $[3]_S$) are shifted up or down by a quantity five times smaller. Then the ground state and the Roper resonance are both lowered while the negative parity states are much less affected by the GBE interaction, leading in practice [3,4] to a good level ordering.

In Ref. [10] it was found that the two-pion exchange plays also a significant role in the quark-quark interaction. The obtained interaction has a spin independent central component, which, averaged over the isospin part of the nucleon wave function, gives rise to an attractive spin independent interaction. The σ-exchange represents in fact a good approximation of such a central interaction. Here we have shown that a scalar attractive interaction cooperates together with the flavour-spin operator (8). In this respect our findings are consistent with those of Ref. [10].

Other important results of Ref. [10], relevant for hadron spectroscopy, are that the spin-orbit components of the two-pion exchange interaction provide a cancelling mechanism for the spin-orbit interaction resulting from the confinement potential and that its tensor component cancels out the one-pion exchange tensor component.

The conclusion is that Goldstone boson exchange models should include explicitly the chiral partner of the pseudoscalar mesons. It might be misleading to mock up its effect in some nontrivial parameters of the model. Our next step will be to study the case of a potential with both scalar and pseudoscalar terms. However the question still remains about the role of the chromomagnetic interaction.

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List of Tables

List of Figures

TABLES

TABLE I. Energies (in MeV) of the first orbitally excited state E_{1p} and the first radially excited state E_{2s} obtained variationally for a $3q$ system described by the Hamiltonian (1) with $\Lambda \to \infty$. The last column gives the values of R defined by Eq. (6).

$g_\sigma^2/(4\pi)$	E_{1p}	E_{2s}	R
0.00	1287	1444	0.688
0.05	1309	1444	0.732
0.10	1333	1444	0.780
0.15	1380	1445	0.871
0.20	1431	1446	0.970

TABLE II. Values of R, Eq. (6), for the nonrelativistic (NR) and the semirelativistic (SR) 2-body equation with the potential (7). The parameters λ, g and μ are dimensionless, as explained in the text

λ	g	μ	NR	SR
0.01	0	-	0.585	0.588
0.865	0	-	0.585	0.628
2.0	0	-	0.585	0.644
0.865	0.2	1.765	0.597	0.655

TABLE III. The expectation value of the interaction (8) for a colour singlet 3q system

state	$\langle O^{IS} \rangle$
$[3]_O[3]_{FS}[21]_F[21]_S$	–15
$[21]_O[21]_{FS}[21]_F[21]_S$	–3
$[21]_O[21]_{FS}[21]_F[3]_S$	3

FIGURES

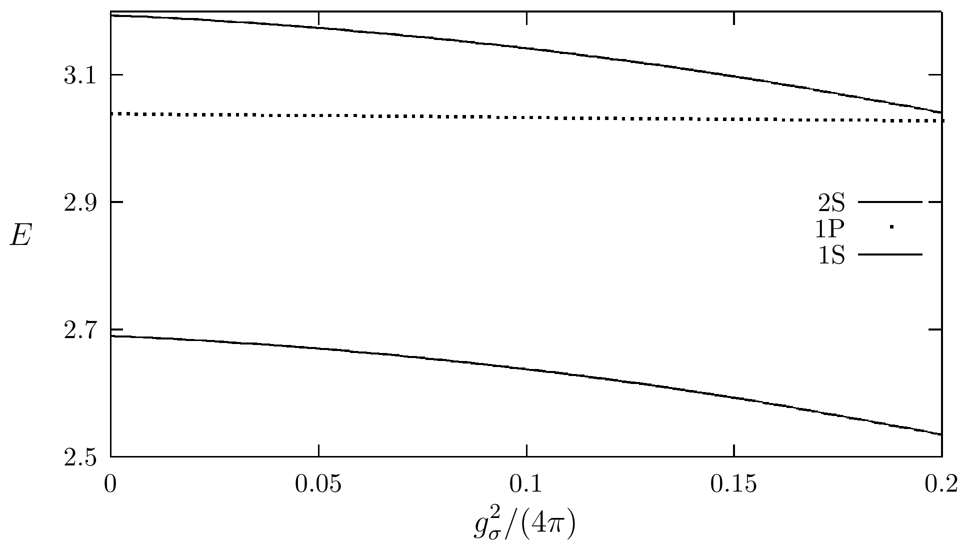

FIG. 1. Energies of the first three eigenvalues of Hamiltonian (1) for increasing values of the coupling constant $g_\sigma^2/(4\pi)$ in the limit $\Lambda \to \infty$.

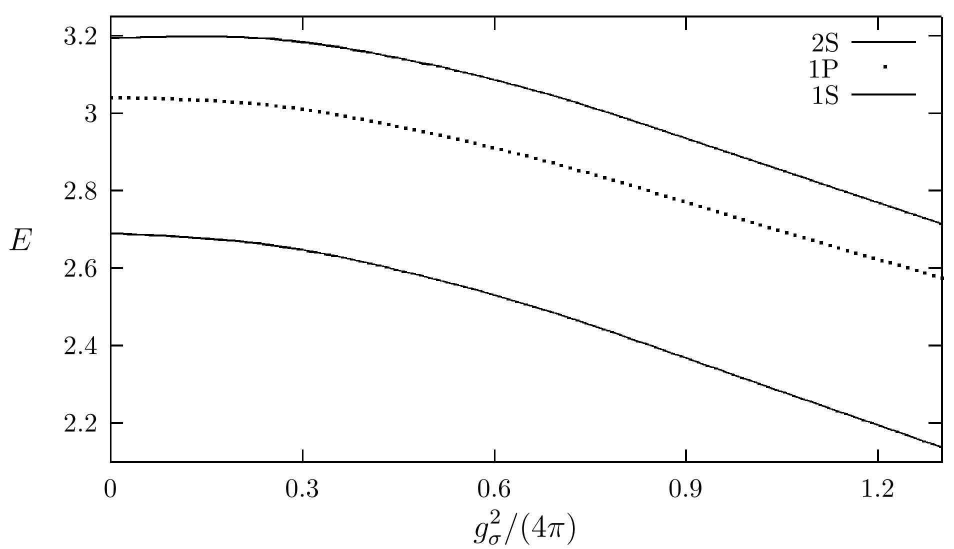

FIG. 2. Same as Fig. 1 but for a finite cut-off $\Lambda = 2$ GeV.